

Analytical solutions of accreting black holes immersed in a Λ CDM model

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Abstract

The evolution of the mass of a black hole embedded in a universe filled with dark energy and cold dark matter is calculated in a closed form within a test fluid model in a Schwarzschild metric, taking into account the cosmological evolution of both fluids. The result describes exactly how accretion asymptotically switches from the matter-dominated to the Λ -dominated regime. For early epochs, the black hole mass increases due to dark matter accretion, and on later epochs the increase in mass stops as dark energy accretion takes over. Thus, the unphysical behaviour of previous analyses is improved in this simple exact model.

1. Introduction

For the past decade, there has been overwhelming evidence of an accelerated expansion of the universe. The simplest model proposed to explain such a phenomenon is a fluid known as *dark energy*, in which the parameters are best fit by Einstein's cosmological constant [1]. Other models have also been proposed, many of which have very interesting cosmological implications, such as quintessence, which proposes a unified fluid with the characteristics of both dark matter and dark energy, as well as more radical models such as phantom (super-negative) energy.

In many cosmological models, the production of primordial black holes is possible by a variety of mechanisms. It is still a matter of debate whether these relics are present [2] and play a role as seeds to the formation of galaxies and galaxy clusters. There may be a substantial evolution of the mass of black holes depending on which epoch they were formed [3, 4] Therefore, it is of interest to have a detailed and accurate description of such astrophysical objects along the evolution of the universe.

In this work we study the evolution of black holes through accretion of perfect fluids, by generalizing the approach to this problem started by the works by Babichev *et al* [5] and based on the covariant conservation of the energy-momentum tensor in the Schwarzschild metric. We consider the particular case

of accretion of dark matter and dark energy as described by the Λ CDM scenario. We find a simple analytical model to describe the mass of the black hole as a function of time which depends solely on the initial values of the energy densities of dark matter and dark energy. We also present specific descriptions of the model for isolated physical species.

2. Solution to the accretion equation

2.1. The Λ CDM scenario

The most important constituents of our present universe are dark energy (denoted with Λ when modeled as a cosmological constant) and cold dark matter (CDM). According to the WMAP5 data [1], these two species are responsible for 95% of the energy content in the universe, leaving the remaining 5% for baryons, neutrinos and radiation. Therefore, it is a sufficiently good approximation to consider a universe containing only dark matter and dark energy, provided we do not go too far into the past epochs, when the contribution from the radiation field was relevant.

To describe the Λ CDM universe, we start with the local energy conservation law of a cosmologically evolving fluid, which states

$$\dot{\rho} + 3H(\rho + p) = 0 \quad (1)$$

so we have $\rho + p = -\dot{\rho}/3H$. From the Friedmann equation $3H^2 = 8\pi G\rho$ we have

$$H = \left(\frac{8\pi G}{3}\right)^{1/2} \rho^{1/2}. \quad (2)$$

Thus, given the equation of state of the fluids, it is possible to find $\rho(a)$ from (1) and then insert the solution on (2) to find $a(t)$ [6].

In this work, we are interested in a universe filled with matter, black holes and a cosmological constant, which represents quite accurately the present state of our universe in the Λ CDM model. Therefore, the energy density of the combined species is $\rho_t = \rho_{\text{DM}} + \rho_{\Lambda}$ (where black holes are assumed never to be dynamically important), which, after inserting their dependency on the scale factor, reads

$$\rho_t = \rho_{\text{DM}}^0 \left(\frac{a}{a_0}\right)^{-3} + \rho_{\Lambda}. \quad (3)$$

The Friedmann equation (2) for such a combination reads

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left[\Omega_{\text{DM}} \left(\frac{a}{a_0}\right)^{-3} + \Omega_{\Lambda} \right] \quad (4)$$

and the solution is

$$a = a_0 \left[\frac{\sinh \left(\frac{3}{2} H_0 \sqrt{\Omega_\Lambda t} \right)}{\sqrt{\frac{\Omega_\Lambda}{\Omega_{\text{DM}}}}} \right]^{2/3}. \quad (5)$$

We use this result to compute the evolution of one test black hole immersed in a universe containing these two species on the following sections.

2.2. The accretion equation

We start with the accretion equation written by Babichev *et al.* [5], based on the conservation of the energy-momentum tensor of a perfect non-self-gravitating fluid in the Schwarzschild metric [7], and a mass variation term which can be justified from geometrical properties of the energy-momentum tensor in diagonal metrics [8]

$$\dot{m} = 4\pi A m^2 [\rho + p(\rho)] \quad (6)$$

where ρ and p are the energy density and pressure of the fluid at infinity.

We neglect the effects of Hawking evaporation, which would otherwise introduce a series of regime transitions from accretion to evaporation [4]. Since evaporation only sets off after a certain temperature threshold, this is equivalent to assuming that black holes are always colder ($T \propto 1/m$) than the cosmic environment.

Equation (6) may be rewritten through a change of variables as

$$\frac{dm}{d\rho} \dot{\rho} = 4\pi A m^2 (\rho + p). \quad (7)$$

After substituting the term $(\rho + p)$ using the conservation equation (1) on (7), we find

$$\frac{dm}{d\rho} \dot{\rho} = -\frac{4\pi A m^2 \dot{\rho}}{3H} \quad (8)$$

with a first integral

$$-\frac{1}{m} = -\left(\frac{8\pi}{3G}\right)^{1/2} \rho^{1/2} + C. \quad (9)$$

To find the integration constant we set the initial value for the black hole mass m_i at the instant with the initial fluid density ρ_i

$$C = \left(\frac{8\pi}{3G}\right)^{1/2} A \rho_i^{1/2} - \frac{1}{m_i}. \quad (10)$$

Inserting the value of the constant on equation (9) we find the black hole mass as a function of the background density [9, 10]

$$m(\rho) = \frac{m_i}{1 + m_i \sqrt{\frac{8\pi}{3G}} A^2 \left(\rho^{1/2} - \rho_i^{1/2} \right)}. \quad (11)$$

We can also find the present value of the mass of a black hole under these conditions by writing (11) in terms of m_0 . Inverting it, we are able to write the initial mass of a black hole as a function of its present mass, provided we know the initial value of the energy density of its main accreted component

$$m_i = \frac{m_0}{1 - m_0 \sqrt{\frac{8\pi}{3G}} A^2 \left(\rho_0^{1/2} - \rho_i^{1/2} \right)}. \quad (12)$$

3. Accretion of cosmological fluids

3.1. Cold dark matter (dust)

For a universe filled only with non-relativistic matter, $\rho_{\text{DM}} \propto a^{-3}$ one obtains the density from the Einstein–de Sitter model

$$\rho_{\text{DM}} = \rho_{\text{DM}}^i \left(\frac{t_i}{t} \right)^2. \quad (13)$$

A black hole accreting matter in this scenario will evolve as

$$m_{\text{DM}}(t) = \frac{m_i}{1 + m_i \sqrt{\frac{8\pi}{3G}} A^2 \rho_{\text{DM}}^i \left[\frac{t_i}{t} - 1 \right]}. \quad (14)$$

This behaviour may seem outrageous, as the mass diverges in a finite time. However, it has been made clear [11, 12, 3] that such a result is an artifact of the local test-fluid approximation which does not take into account the back-reaction onto the black hole metric, the consequence being the absence of an upper bound for the accretion of a pressureless fluid. Therefore, it cannot be used at all for arbitrarily high dark matter densities and time intervals.

On the low density limit, however, this approach may prove useful as a mechanism to gauge the importance of different components to the black hole mass growth. In fact, if one carries out these calculations for realistic initial values of the dark matter density in the matter-dominated era, it can be seen that the mass growth is rapidly quenched by the cosmic evolution of the dark matter component and never becomes important [4].

3.2. Dark energy only

A universe filled with dark energy which behaves as a cosmological constant will evolve according to the “pure” de Sitter model, $a \propto e^{Ht}$. Such a component with an equation of state $p_\Lambda = -\rho_\Lambda$ has a constant energy density, as can be seen from equation (1). Therefore, according to equation (11), a black hole immersed in such a fluid remains at a constant mass m_Λ .

Although the analysis from section 2.2 cannot be applied to the cosmological constant due to the $\dot{\rho} = 0$ term in (7), the result is still consistent with the results from Babichev *et al.* [5] and the constant mass parameter employed to derive the Schwarzschild–de Sitter metric, as expected.

3.3. Dark energy and cold dark matter (Λ CDM)

In a universe with a scale factor evolving as (5), a black hole will accrete the total energy, so we must use $\rho_{\text{tot}} = \rho_{\text{DM}} + \rho_{\Lambda}$ in equation (11). Therefore, the matter density will evolve as

$$\rho_{\text{DM}}(t) = \rho_{\text{DM}}^i \frac{\Omega_{\Lambda}}{\Omega_{\text{DM}}} \frac{1}{\sinh^2 \left(\frac{3}{2} H_0 \sqrt{\Omega_{\Lambda}} t \right)} \quad (15)$$

and the dark energy density will remain constant, so the mass will follow the following function of time, with $\rho_{\text{DM}}(t)$ given by (15)

$$m_{\text{tot}}(t) = \frac{m_i}{1 + m_i \sqrt{\frac{8\pi}{3G}} A^2 \left\{ [\rho_{\Lambda} + \rho_{\text{DM}}^i]^{1/2} - [\rho_{\Lambda} + \rho_{\text{DM}}(t)]^{1/2} \right\}}. \quad (16)$$

A close inspection to equation (16) shows that, if one takes an arbitrarily high initial value for the dark matter density, one recovers the unphysical Einstein–de Sitter evolution from equation (14). Therefore, to avoid the same artifacts due to neglecting the back-reaction, the initial time and asymptotic densities must be set close enough to the dark energy-dominated era, when the black hole is massive enough and the evolution of the scale factor is fast enough to dilute the dark matter density. This requirement is as important as neglecting the Hawking evaporation, which also requires a sufficiently large mass.

Figure 1 shows the evolution of the black hole masses on the scenarios discussed above.

4. Conclusion

In this paper we have studied the evolution of black holes which interact with a combination of dark matter and dark energy, considering that in the vicinity of the black hole the metric is of Schwarzschild type. This model represents a quasi-static accretion onto a single black hole. We found the black hole mass as a function of time by finding the energy densities of dark matter and dark energy as solutions to the Friedmann equation and the local conservation law, in which the black hole fluid is just a negligible fraction of the dark matter content.

It is worth mentioning that the derivation of equation (11) does not take into account the back-reaction of the accretion on the surrounding metric, since it only considers a test fluid. Therefore, some inaccuracies may arise when large m_i and large variations of it are computed. On the other hand, the considered black holes cannot be assumed too small either, since the accretion condition implies a flux of energy into them instead of the opposite physical situation represented

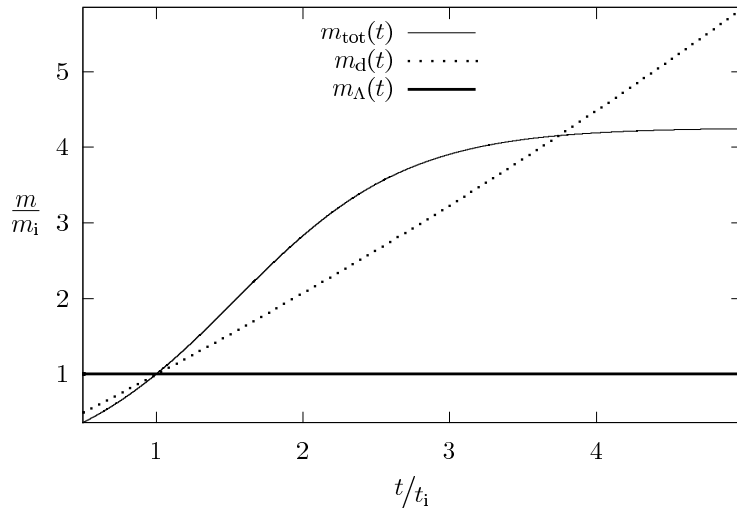


Figure 1: Evolution of a black hole with an initial mass of $m_i = 10^{-3}m_\odot$ in the three scenarios discussed on section 3. Notice that accretion becomes negligible for $m_{\text{tot}}(t)$ as dark energy dominates.

by the Hawking evaporation [13, 14, 15]. To achieve a more accurate description for a wider range of conditions, one must consider the surrounding fluid also as self-gravitating, and the full Einstein equations must be solved.

From the results presented on section 3, we conclude that black holes immersed in such a scenario feature an increase in mass, but such an increase stops when dark energy becomes the most important component. These results are consistent with the calculations performed by Babichev *et al.* [5], and they also further generalize their model by taking into account the evolution of the surrounding fluid with time.

Further developments of this approach must consider other types of dark energy candidates, such as quintessence and quartessence, as well as a more complete combination of fluids, which should provide a clearer knowledge on the evolution of primordial black holes and their influence on cosmology in general.

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